

Relativistic Ground and Excited State Energies of a A -Particle in Hypernuclei Using Woods-Saxon Potentials

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The relativistic Dirac equation with a scalar potential and the fourth component of a vector potential of the Woods-Saxon shape is solved numerically for potential parameters obtained by a last squares fitting procedure of the ground state binding energies of the A in a number of hypernuclei and its binding energies in the ground and excited states (as well as the relevant spacings) for various hypernuclei are determined. The results are in very good agreement with the preliminary experimental ones given by Chrien on the basis of the (π^+, K^+) reaction on nuclei.

1. Introduction

Our knowledge about the binding energies in the ground and excited states of a A -particle in hypernuclei was more limited a few years ago [1] than it is now. In recent (π^+, K^+) experiments these energies have been determined for a number of hypernuclei ranging from ${}^9\text{Be}$ to ${}^{89}\text{Y}$ [2]. These experimental results can be analysed by means of non-relativistic quantum mechanics [3, 4]. Relativistic approaches have also been undertaken [5–11].

In this paper we adopt the Dirac equation and assume an average A -nucleus potential made up by an attractive part U_s and a repulsive part U_v , both of the Woods-Saxon type. We determine the parameters of these potentials by a least squares fitting procedure [11]. The calculated binding energies in the states $1s$, $1p$, $1d$, $1f$, $1g$ for various hypernuclei ranging from ${}^9\text{Be}$ to ${}^{208}\text{Pb}$ are then compared with the experimental ones given by Chrien [2]. We also calculate the energy splittings Δ_{sp} between the various states and compare them with the results of other workers [3, 12, 13].

2. Basic Formalism

We assume that the motion of the A -particle in hypernuclei is governed by the Dirac equation [5–7, 9–11]

$$\{c \mathbf{a} \cdot \mathbf{p} + \beta \mu c^2 + \beta U_s(r) + U_v(r)\} \psi = E \psi, \quad (1)$$

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where $\mathbf{a} = (\alpha_1, \alpha_2, \alpha_3)$ and β are Dirac matrices, ψ is the Dirac eigenfunction and E the total energy, i.e.

$$E = \varepsilon + \mu c^2 = -B_A + \mu c^2,$$

where B_A is the binding energy of the A particle in the hypernucleus. The average A -nucleus potential was constructed by means of an attractive scalar relativistic single-particle potential $U_s(r)$ (generated by a scalar boson exchange interaction) and a repulsive relativistic single particle potential $U_v(r)$, which is the fourth component of a vector potential (resulting from the fourth component of a vector boson exchange interaction). From (1) one obtains the following radial equation of the Schrödinger type [5–7, 10]:

$$g''(r) - \left\{ \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} (V_{\text{centr}}(r, B_A) + V_{\text{s.o.}}(r, B_A) + B_A) \right\} g(r) = 0, \quad (2)$$

where

$$\begin{aligned} V_{\text{centr}}(r, B_A) &= U_+(r) + \frac{\hbar^2}{2\mu} \left\{ \frac{1}{\hbar^2 c^2} (U_+(r) + B_A)(U_-(r) - B_A) \right. \\ &\quad \left. - D^{-1}(r) D'(r) r^{-1} + (2D(r))^{-1} D''(r) \right. \\ &\quad \left. + \frac{3}{4} D^{-2}(r) (D'(r))^2 \right\}, \end{aligned} \quad (3)$$

$$V_{\text{s.o.}}(r, B_A) = -\frac{\hbar}{2\mu} \frac{1}{(2\mu c^2 - B_A + U_-(r))} \frac{1}{r} \frac{dU_-(r)}{dr} I \sigma, \quad (4)$$

$$U_{\pm}(r) = U_s(r) \pm U_v(r) \quad (5)$$

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and

$$D(r) = \frac{1}{hc} (2\mu c^2 - B_A + U_-(r)). \quad (6)$$

(Our notation is the same as that of [5–7] and [9–11].

3. Numerical Results and Comments

Assuming that the potentials $U_{\pm}(r)$ are of the Woods-Saxon shape

$$U_{\pm}(r) = \frac{-D_{\pm}}{1 + e^{(r-c)/a}}, \quad (7)$$

where c is given either by the simple expression [14]

$$c = r_0 A^{1/3} \quad (8)$$

or by the more complicated one [15]

$$c = (1/2)^{1/3} r_0 A^{1/3} \{ [1 + (1 + 2^2 3^{-3} (\pi a/r_0 A^{1/3})^6)^{1/2}]^{1/3} + [1 - (1 + 2^2 3^{-3} (\pi a/r_0 A^{1/3})^6)^{1/2}]^{1/3} \}, \quad (9)$$

(where $A = A_{\text{core}}$), we have solved the differential equation (2) numerically for the potential parameters D_+ , D_- , r_0 and a (determined by a least squares fitting procedure of the ground state binding energy of the Λ in a number of hypernuclei) [11], and we have found the binding energies of the Λ -particle in the 1s, 1p, 1d, 1f, 1g states in various hypernuclei. These results are analysed and commented on below. Our treatment being relativistic, the excited states are splitted up naturally into two states, and thus we give the binding energies of the states 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 1d_{3/2}, 1f_{7/2}, 1f_{5/2}, 1g_{9/2}, 1g_{7/2} for various hypernuclei. Since the experimental resolution is of the order of 2 to 3 MeV and the splitting of the levels (due to the spin orbit coupling) is much smaller (around 0.1 to 0.2 MeV for the heavier hypernuclei and around 1 MeV for the lighter ones), we give also the binding energy of the unsplit states as the average of the corresponding splitted ones, namely

$$B_A(P) = \frac{B_A(p_{3/2}) + B_A(p_{1/2})}{2}, \quad (10)$$

and similarly for the d, f and g states.

In order to see comparison as how much the potential $V_{\text{S.O.}}(r, B_A)$ (i.e. the spin orbit part) affects the binding energy of the states p, d, f and g we have calculated also the binding energy of the unsplit states using only the central part of the potential, namely

$V_{\text{centr}}(r, B_A)$ (that is by taking $V_{\text{S.O.}}(r, B_A) = 0$). These states are denoted by p_c , d_c , f_c and g_c . As can be seen from our tables the binding energies of the states p_c , d_c , f_c , g_c differ very little from the binding energies of the states p , d , f and g , the difference being of the order of 0.1 MeV for the lighter hypernuclei and practically zero for the heavier ones. Since experimentally only the states p , d , f , g can be recognized due to an experimental resolution of the order of 2 to 3 MeV, we see that is not easy to deduce from the (π^+, K^+) experiments the spin orbit coupling in hypernuclei.

Also in this work we give the spacing between the various levels, namely

$$\Delta_{\text{sp}} = B_A(s) - B_A(p) \quad (11)$$

and Δ_{pd} , Δ_{df} , Δ_{fg} defined analogously.

We have performed calculations with five sets of potential parameters:

$$D_+ = 29.8 \text{ MeV}, r_0 = 1.198 \text{ fm}, D_- = 300 \text{ MeV (fixed)}, \\ a = 0.6 \text{ fm (fixed)}, \chi^2 = 21.254,$$

$$D_+ = 30 \text{ MeV}, r_0 = 1.211 \text{ fm}, D_- = 443 \text{ MeV (fixed)}, \\ a = 0.6 \text{ fm (fixed)}, \chi^2 = 21.675,$$

$$D_+ = 28.3 \text{ MeV}, r_0 = 1.45 \text{ fm}, D_- = 443 \text{ MeV (fixed)}, \\ a = 0.6 \text{ fm (fixed)}, \chi^2 = 12.009,$$

$$D_+ = 29.5 \text{ MeV}, r_0 = 1.153 \text{ fm}, D_- = 417 \text{ MeV}, \\ a = 0.32 \text{ fm}, \chi^2 = 14.733,$$

$$D_+ = 28.2 \text{ MeV}, r_0 = 1.475 \text{ fm}, D_- = 427 \text{ MeV}, \\ a = 0.63 \text{ fm}, \chi^2 = 12.01.$$

For each set the corresponding value of χ^2 (goodness of fit criterion) is indicated. The potential parameters were obtained by a least squares fitting procedure, and the parameters which were kept fixed in the fit are indicated by the word “fixed”. The values of the fixed parameters were inferred from other studies, namely $D_- \cong 443 \text{ MeV}$ from [8], $D_- \cong 300 \text{ MeV}$ from [7], and $a = 0.6 \text{ fm}$ from [16].

The set of parameters which we think gave on the whole the best agreement with the experimental results yielded the values given in Table 1. Corresponding tables for the other four sets of parameters can be obtained from the authors on request.

Figure 1 shows our results of the binding energies of the Λ particle in hypernuclei in the states 1s, 1p, 1d, 1f, 1g given in Table 1 fit the preliminary experimental results of Chrien [2]. As it is seen, the agreement is very good. Figure 2 shows how the spacings Δ_{sp} and Δ_{pd} of various hypernuclei given in Table 1 fit the experimen-

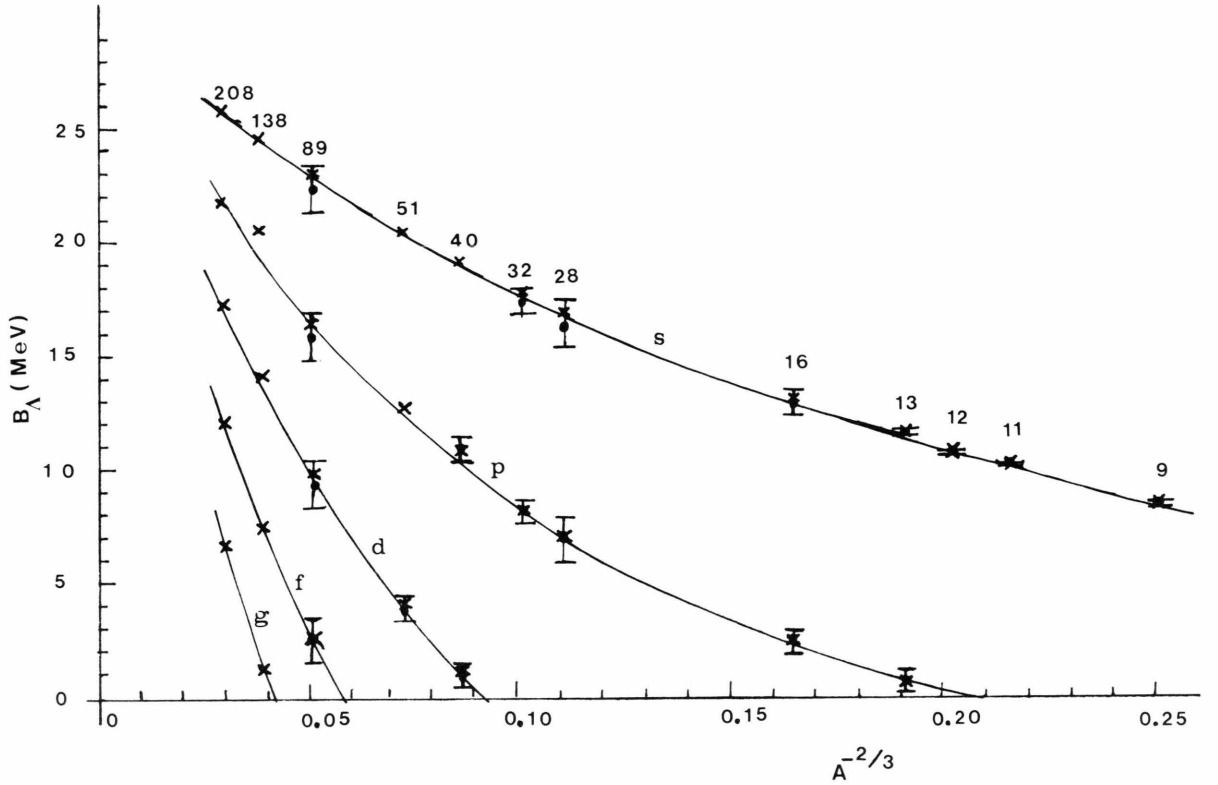


Fig. 1. The ground state and excited state binding energies of the Λ particle in hypernuclei calculated from (2) with the potential parameters $D_+ = 29.8$ MeV, $r_0 = 1.198$ fm, $D_- = 300$ MeV (fixed), $a = 0.6$ fm (fixed), using $c = r_0 A_{\text{core}}^{1/3}$ versus $A^{-2/3}$ (crosses) and the preliminary experimental binding energy of various hypernuclei given by Chrien [2] (points with error bars).

Table 1. Binding energies (in MeV) of the ground and excited states as well as spacings between the (averaged) states for a Λ particle in hypernuclei and for a number of hypernuclei obtained from (2) using the following potential parameters: $D_+ = 29.8$ MeV, $r_0 = 1.198$ fm, $D_- = 300$ MeV (fixed), $a = 0.6$ fm (fixed), $\chi^2 = 21.254$. For the radius c expression (8) was used.

A_{core}	Hyper-nuclei	$s_{1/2}$ B_A	$p_{3/2}$ B_A	$p_{1/2}$ B_A	p B_A	p_c B_A	Δ_{sp}	$d_{5/2}$ B_A	$d_{3/2}$ B_A	d B_A	d_c B_A	Δ_{pd}	$f_{7/2}$ B_A	$f_{5/2}$ B_A	f B_A	f_c B_A	Δ_{df}	$g_{9/2}$ B_A	$g_{7/2}$ B_A	g B_A	g_c B_A	Δ_{fg}
8	${}^9_{\Lambda}\text{Be}$	8.6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10	${}^{11}_{\Lambda}\text{B}$	10.3	0.1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
11	${}^{12}_{\Lambda}\text{C}$	11.0	0.5	—	—	0.3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	${}^{13}_{\Lambda}\text{C}$	11.6	1.1	0.3	0.7	0.8	10.9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15	${}^{16}_{\Lambda}\text{O}$	13.2	2.7	1.8	2.2	2.4	11.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
27	${}^{28}_{\Lambda}\text{Si}$	17.1	7.7	6.9	7.3	7.4	9.8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
31	${}^{32}_{\Lambda}\text{S}$	18.0	8.9	8.2	8.5	8.6	9.5	0.2	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39	${}^{40}_{\Lambda}\text{Ca}$	19.3	10.9	10.3	10.6	10.7	8.7	2.3	1.4	1.8	1.9	8.8	—	—	—	—	—	—	—	—	—	—
50	${}^{51}_{\Lambda}\text{V}$	20.6	12.9	12.5	12.7	12.8	7.9	4.8	3.9	4.4	4.4	8.3	—	—	—	—	—	—	—	—	—	—
88	${}^{89}_{\Lambda}\text{Y}$	23.2	17.2	16.9	17.0	17.1	6.2	10.4	9.8	10.1	10.2	6.9	3.3	2.4	2.9	2.9	7.3	—	—	—	—	—
137	${}^{138}_{\Lambda}\text{Ba}$	24.7	19.9	19.8	19.9	19.9	4.8	14.4	14.0	14.2	14.2	5.7	8.3	7.6	7.9	8.0	6.3	1.9	0.9	1.4	1.4	6.5
207	${}^{208}_{\Lambda}\text{Pb}$	25.8	22.1	22.0	22.0	22.0	3.8	17.6	17.3	17.4	17.5	4.6	12.5	12.0	12.2	12.3	5.2	7.0	6.2	6.6	6.6	5.6

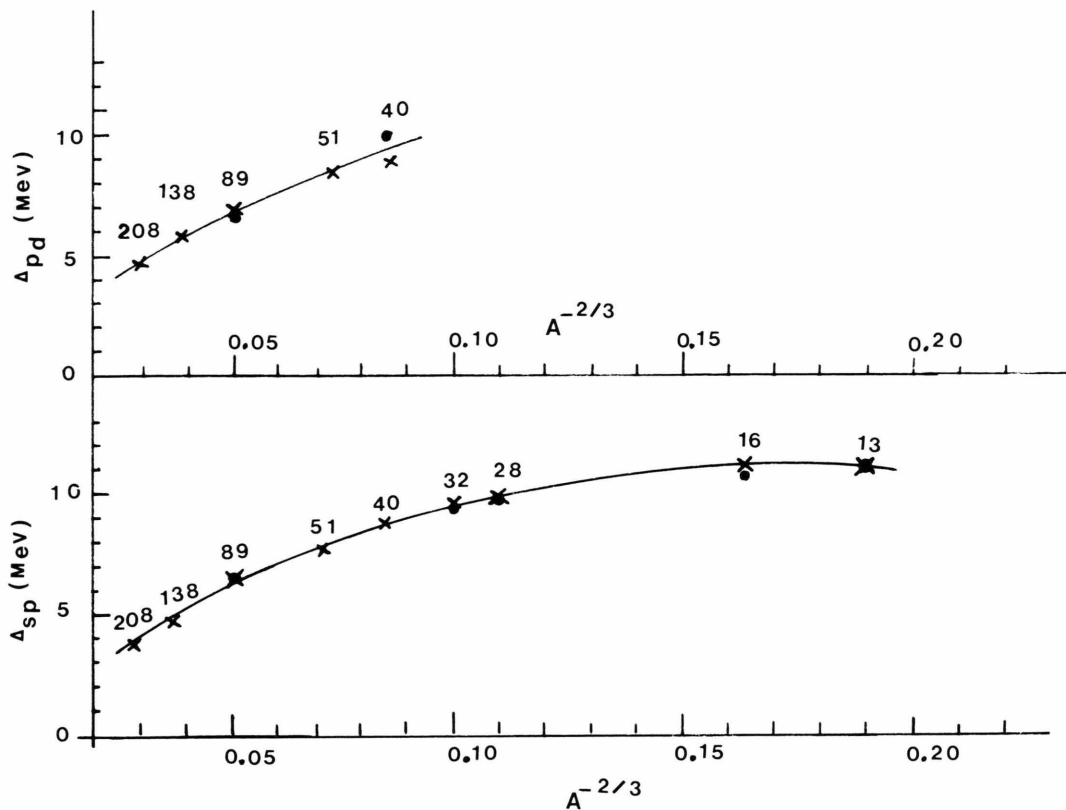


Fig. 2. Spacings Δ_{sp} and Δ_{pd} appearing in Table 1 plotted versus $A^{-2/3}$. The calculated values are given by crosses and the experimental ones by points.

tal data. It is seen again that the agreement is very good.

Dover [3] gave similar curves using the non-relativistic approach with the Woods-Saxon potential and with a different expression for c . Non-relativistic calculations concerning the oscillator spacing $\hbar\omega_A = \Delta_{sp}$ for a Λ in hypernuclei were performed also in [12, 13, 17, 18]. The results of these authors can somehow be compared to the Δ_{sp} spacing obtained by us though

the comparison is not done on the same footing since our treatment is relativistic.

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